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Application of Mathematical Models in Agriculture- A Review

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Abstract

Application of mathematical models are for solving problems in agriculture for a scientific understanding, quantitative expression and to take strategic decisions. Mathematical models include mechanistic, empirical, deterministic, and stochastic approaches. It has dynamic models with differential equations, static models with algebraic for a specific set of conditions, deterministic models suggest solutions, stochastic model deals with defined by probability functions, mechanistic model deals with theory or hypothesis, and empirical models uses existing data to explain the relationship between one or two variables. Mathematical models have been developed to investigate specific issues limited to mathematical formulation and the added complexity inherent of integrated models. Mathematical methods of resource utilization optimization have been used in practice and the first mathematical programming approaches include the method of linear programming (simplex method). Linear approach to modeling establishes the relationship between a dependent variable and one or more independent variables. In the linear equation, dependent and independent variables, coefficients, intercept or the bias coefficient and degree of freedom have been used. The application of mathematical models in agriculture portrays the main methods of various mathematical tools like analytical, simulation and empirical. This paper aims at application of Mathematical Models in agriculture.

Keywords: Mathematical models, mechanistic, empirical, deterministic, stochastic approaches, dependent and independent variables, linear programming, Linear approach, agriculture.

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Introduction

Mathematical models are used for solving the intricate issues and problems in the agriculture for a number of reasons like developing scientific understanding, quantitative expression of a system for decision making, to take a tactical decision as well as strategic decisions by the agricultural planners. This study aims at application of Mathematical Models in agriculture. It is because farm planning in agriculture is imperative in different size of farm holdings to achieve the desired results. Agriculture and its allied sectors are pervasive in terms of employment and livelihoods for the marginal, small and large farmers, who dominate the agriculture in countries like India. To attain the Sustainable Development Goals (SDGs) in ending poverty and bringing an inclusive growth, activities related to agriculture need to be closely integrated with the SDG targets. As a result, Indian government has taken many initiatives to ensure its long-term development of agriculture. Various government interventions for the growth of allied sectors such as animal husbandry, dairying, and fisheries demonstrate the commitment to maximising the potential of allied sector to enhance farm welfare. Aside from several initiatives aiming at enhancing agricultural productivity and marketing, the government also implements a large food management programme with significant financial implications in terms of food subsidies. Agriculture and its allied sector sectors employ 54.6 percent of the workforce (Census 2011) and account for 17.8 percent of the India's Gross Value Added in 2019-20.

Mathematical models consist of mechanistic and empirical with deterministic and stochastic approaches. The mechanistic model deals with one level in the hierarchy by considering processes of mechanisms through which changes occur at lower levels. The empirical model account for mechanism by which changes occur in the system. In deterministic approach, the empirical model attempts to perform prediction from a regression relationship while mechanistic deals with Newtonian mechanics with different equations. In Stochastic model, for empirical study tools like Analysis of variance is used, while for mechanistic approach Mendelian inheritance is applied with probabilistic equations.

In a comprehensive crop model, yield prediction has depended on statistical regression models, sometimes improved by accounting for the soil moisture balance (Baier & Robertson, 1968) or by calculating crop transpiration with simplified procedures (Zaban, 1981). A tulip bulb model uses intermediate harvests to update the yield prediction (Benschop, 1985) but the model has not been applied in practice.

Reddy, V.R., D.N. Baker, F.D. Whisler and R.E. Fye, (1987) have used a model to examine cotton yields in the U.S.A since 1965 with a threefold increase from 1935 to 1965. The model found damage in root function due to use of herbicide which caused for the decline in yield. The study found that use of herbicide damage to roots and consequent yield reduction experimentally over a period of 20 years.

Baldwin (1995) describes that 'dynamic models are with differential equations; static models have algebraic and solved for a specific set of conditions; deterministic models suggests solutions of an equation or set of equations; stochastic model deals with defined by probability

functions that seeks to take account of the variance that is not fully understood; mechanistic model provides equations that is derived from theory or hypothesis about the fundamental nature of the system; mechanistic model refers with a casual relationships within the system related to a broad range of realities; and empirical models uses existing data to explain the relationship between one or two variables (Riggs, 1963).

According to J. H.M. Thornley, (1996) developed “water sub-model is designed for use with plant growth simulators that represent internal plant substrates and variable root partitioning. The model calculates water row from soil to root, root to shoot, and shoot to the atmosphere, for a closed-canopy situation. The model has three state variables: the masses of water in the soil, root and shoot, and represents the processes of evapotranspiration, rainfall interception and evaporation from the canopy, and drainage. The Penman Monteith equation is used for crop transpiration. The fluxes of water from soil to root, and root to shoot, are driven by water potential difference. Tissue water potential and its components are calculated from tissue water content and other plant variables and parameters. The model is able to simulate osmoregulation and describes a variable relationship between tissue water potential, its components and relative water content, depending on growth conditions. The model has elsewhere been integrated with two plant ecosystem models: for grassland and forest”.

D. Dourado-Neto, D. A. Teruel, K. Reichardt, D.R. Nielsen, J. A. Frizzzone, And O.O.S. Bacchi (1998) have observed that the mathematical models are used for agricultural purposes in order to forecast the results produced by a given system, effects of certain environmental conditions and agricultural practices on crop performance. They have opined that ‘modelling represents a better way of synthesizing knowledge about different components of a system, summarizing data, and transferring research results to users.’

R. Bixby, M. Fenelon, Z. Gu, E. Rothber, and R. Wunderling (2000) have observed that due to NP-hardness of supply chain problems and the large sized data widely used in the real world are meta-heuristics like genetic algorithm and particle swarm optimization. In optimization problems, the solution technique of mixed integer programming models, or mixed integer linear programming, are employed in supply chain problems.

J. H. M. Thornley and J. France (2007), are of the view that in agriculture and science simple mathematical models like hierarchy and description have been applied. Hierarchy, description, empirical, mechanistic and teleonomic models are the different model types which requires objectives that are ‘critical in a modelling project, models for research are usually different from models for application, and the benefits from a model can be diverse’.

O.D. Sirotenko (2009) has discussed four aspects viz., ecological, energetic, and productive. With these aspects in empirical models is to describe the main objective of mechanistic simulation to explain the described one. In this, the sense of a concept of hierarchy level is explained by an exemplotypical of agriculture:

Level	Description
$i+1$	Totality of organisms (crops, herd)
i	Organism (animal, plant)
$i-1$	Organs
.....	Tissues
.....	Cells

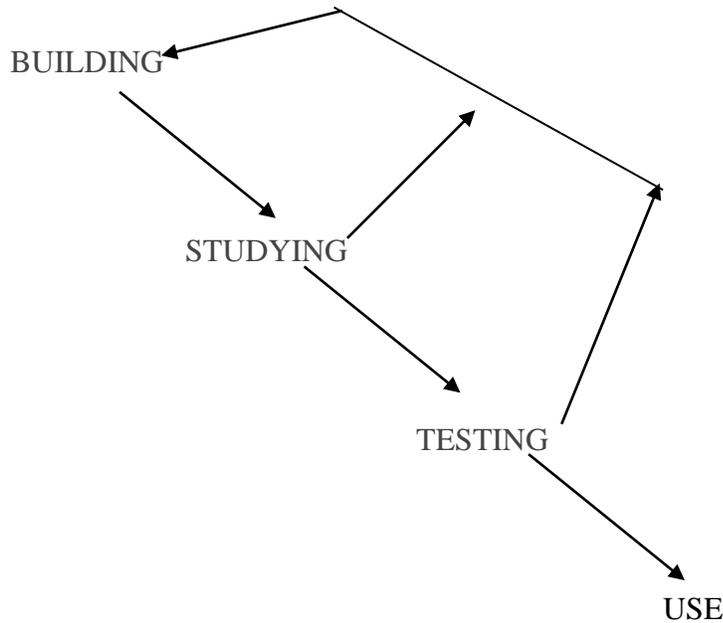
According to him. in the static mathematical model while construction no time variable is involved and the vegetation period is divided into a large number of time intervals that increases the number of independent variables and, consequently, decreases the accuracy of model coefficient assessment.

M.J. McPhee (2009) mathematical modelling frameworks with key features are required to understand the complexity and diversity in agricultural production and marketing systems. Biology, computer programming, statistics, mathematics, economics, and social science needs such models. Mathematical models have been developed to investigate specific issues such as the effects of climate change, climate variability, and to assist third world countries improve their production at the farm gate.

W. Zhang, W.E. Wilhelm, (2011) mathematical models have remained relatively small and within the reach of commercial solvers as the industry grows and problems become larger, basic research will become increasingly important to ensure solvability. They emphasized to consider mixed integer programming models as solution approach for fresh agri-food supply chain problems.

N.Mason, H. Flores, J.R. Villalobos, and O. Ahumada, (2015) have opined that the technology and tools for increasing the efficiency of agri-food supply chain have been researched with 'limited mathematical formulation, which contrasts with the intuition of traditional decision-makers, their limitations on capturing the whole system dynamics, and the added complexity inherent of integrated models. Therefore, this work aims to focus on research in fresh agri-food supply chains which employ mixed integer programming models as solution approach to investigate its improving potential especially in the efficiency of the products delivery'.

G.Marion and Lawson (2015) have categorised four broad categories of modelling like building, studying, testing, and use as given in the following sequencing type. Modelling project begins from building through the use. Defects are found at the studying as well as testing stages which are corrected by revolving back to building stage. Such process will keep on going until perfection is reached in the relevant model building.



They have developed a deterministic model which describes such a population in continuous time is the differential equation as given below:

$$\frac{dp}{dt} = ap$$

where $p(t)$ is population size at time t , and a is a constant. Solution of this equation by integration gives

$$p(t) = p(0)e^{at}$$

where $p(0)$ is population size at time zero. According to this solution, populations grow in size at an exponential rate.

O.D. Sirotenko and V.A. Romanenkov (2012) paper pronounces an approach for solution to the problems using mathematical modelling and computer technology approaches where the optimization problems are generalized for land, fertilizer, irrigation water, facilities, labour use in plant-growing, the herd structure and feeding rations in livestock. Mathematical methods of resource utilization optimization have been used in practice and the first mathematical programming approaches include the method of linear programming (simplex method). The models of agricultural processes and the optimization methods have been developed quickly based on the specific problem or the specific parameters a wide range of methods from classical analytical studies of the systems of here that all parameters in the economic-mathematical model (resources, technical economic coefficients and coefficients of the objective function) are deterministic, a priori known quantities. Problems considering the risk and uncertainty factors, in which some (and in the general case all) parameters are random values, are known as stochastic problems of resources allocation. Stochastic is the integral attribute of agricultural production

connected with the uncertain character of the environment and, primarily, weather conditions. Consider the statement of a general problem of linear programming in a matrix form: find

$$F(x) = C * X \rightarrow \max \text{ at } AX < B, X > 0.$$

Where the matrix A and the vectors B and C are deterministic, it is the most frequently encountered statement of a problem. In stochastic problems A , B and C may be random. Problems of stochastic programming differ considerably in the objective function.

P.Paam, R. Berretta, M. Heydar, R.H. Middleton, R. García-Flores, and P. Juliano (2016) made a comprehensive and structured review on recent studies in the field of agribusiness planning models, aiming to optimize fresh agri-food supply chain, with a focus on loss minimization in the fruits and vegetables. R.D. Kusumastuti, D.P. Donk, and R. Teunter, (2016) discussed the literature on crop related agrichains focusing on the integration, or lack thereof, harvesting and processing planning and related inventory control issues.

Pia Ghoshal and Bhaskar Goswami (2017) study is based to analyze production efficiency of agricultural system in four regions of India. The paper used a stochastic frontier model, employing time series data ranging from 2005-06 to 2013-14. They have used Battese and Coelli (1995) inefficiency frontier model for panel data is as follows:

$$Y_{it} = \exp(X_{it} \beta + V_{it} - U_{it}) \dots (1)$$

where, Y_{it} , denotes the production at the t -th observation ($t = 1, 2, \dots, T$) for the i -th firm ($i = 1, 2, \dots, N$)

X_{it} , is a $(1 \times k)$ vector of values of known functions of inputs of production and other explanatory variables associated with the i -th firm at the t -th observation;

β is a $(k \times 1)$ vector of unknown parameters to be estimated;

the V_{it} s are assumed to be iid $N(0, \sigma^2 V)$ random errors, independently distributed of the U_{it} s.

the U_{it} are non-negative random variables, associated with technical inefficiency of production, which are assumed to be independently distributed, such that U_{it} is obtained by truncation (at zero) of the normal distribution with mean, $z_{it}\delta$ and variance, σ^2 . z_{it} is a $(1 \times m)$ vector of explanatory variables associated with technical inefficiency of production of firms over time; and

δ is an $(m \times 1)$ vector of unknown coefficients.

Equation (1) specifies the stochastic frontier production function in terms of the original production values. The technical inefficiency effect, U_{it} , in the stochastic frontier model (1) could be specified in equation (2),

$$U_{it} = W_{it} \delta$$

where, the random variable, W_{it} , is defined by the truncation of the normal distribution with zero mean and variance, σ^2 , such that the point of truncation is $-z_{it}\delta$ i.e. $W_{it} > -z_{it}\delta$. These

assumptions are consistent with U_{it} being a non-negative truncation of the $N(z_{it} \delta, \sigma^2)$ -distribution.

They have the production function of the following form:

$$\ln(Y_{it}) = \beta_0 + \beta_1 \ln(INSCRE_{it}) + \beta_2 \ln(NIA_{it}) + \beta_3 \ln(CONFER_{it}) + \beta_4 \ln(CONPES_{it}) + V_{it} - U_{it} \dots (3)$$

where the technical inefficiency effects are assumed to be defined by,

$$U_{it} = \delta_0 + \delta_1 \ln(RATELIT_{it}) + \delta_2 \ln(RATETECHEDU_{it}) + \delta_3 \ln(LENROAD_{it}) + \delta_4 \ln(SHARENSDP_{it}) \dots (4)$$

Where \ln denotes the natural logarithm (i.e. logarithm to the base e);

Y is the total agricultural production of the individual states considered⁴.

$INSCRE_{it}$ represents institutional credit which comprises of purpose wise refinance disbursements by NABARD under investment credit provided to each representative states. It shows refinances given for the purpose of minor irrigation, land development and farm mechanization. It is measured in terms of rupees lakh.

NIA_{it} is the Net Irrigated Area of each state. It is measured in terms of '000 hectares.

$CONFER_{it}$ represents consumption of fertilizers by each representative state. Its principal components include N (nitrogen), P (Phosphate) and K (potassium). It is measured in terms of '000 tonnes⁷.

$CONPES_{it}$ represents consumption of pesticides. It is measured in terms of metric tonnes⁸.

$RATELIT_{it}$ represents rate of literacy of the rural areas of the representative states and the rate is calculated in terms of total rural population of the state.

$RATETECHEDU_{it}$ represents rate of technical education of the rural areas of the representative states and the rate is calculated in terms of total rural population of the state.

$LENROAD_{it}$ represents length of roads per square kilometre area of the representative state. Importance of infrastructure in explaining inefficiency is brought into the analysis by considering this variable.

$SHARENSDP_{it}$ is share of agricultural Net State Domestic Product to total Net State Domestic Product. We have attempted to consider the significance of agricultural sector in the state's economic scenario by this variable.

V_{it} and U_{it} are as defined in the previous section. The V_{it} s are assumed to be iid $N(0, \sigma^2)$ random errors, independently distributed of the U_{it} s. The U_{it} s are non-negative random variables, associated with technical inefficiency of production, which are assumed to be

independently distributed, such that U_{it} is obtained by truncation (at zero) of the normal distribution with mean, $z_{it}\delta$ and variance, σ^2 .

z_{it} is a $(1 \times m)$ vector of explanatory variables associated with technical inefficiency of production of firms over time; and δ is an $(m \times 1)$ vector of unknown coefficients.

The study found a positive relationship between agricultural productivity and higher use of fertilizers, and pesticides is seen along with the significance of net irrigated area and institutional credit for these regions of the selected states.

Linear Regression and its Mathematical implementation

Linear approach to modeling establishes the relationship between a dependent variable and one or more independent variables viz., simple linear regression and multiple linear regression respectively. In the linear equation, dependent and independent variables, coefficients, intercept or the bias coefficient and degree of freedom have been used. Here, multiple linear regression of the following form is used:

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5 + \beta_6 X_6 + \beta_7 X_7 + \mu$$

Where Y is the total food grains per hectare in kg

X_1 = Net area sown in hectare

X_2 = Gross Sown Area in hectare

X_3 = Net Irrigated Area in hectare

X_4 = Gross Irrigated Area in hectare

X_5 = Area under high yield varieties in hectare

X_6 = Consumption of Fertilizer (N+P+K) (lakh tonnes)

X_7 = Consumption of Pesticides (Technical Grade Materials) ('000 tonnes)

β_0 = Constant term

β_1 to β_7 are predictors that has a corresponding slope coefficient

μ represents all the other variables that have not been included as independent variables

The above model is used for the following data and the results and diagrammes are given after the Table 1.

Table 1: Food grains production (in kg. per hectare), area under cultivation (in hectare), consumption of NPK (in lakh tonnes) and pesticides (in '000 tonnes)

Year	Total Foodgrains	Net area sown	Gross Sown Area	Net Irrigated Area	Gross Irrigated Area	Area under high yield varieties	Consumption of Fertilizer (N+P+K) (lakh tonnes)	Consumption of Pesticides (Technical Grade Materials) ('000 tonnes)
2020-21	2386	1452	2089	722	1268	0	321	5503
2019-20	2343	1436	2056	702	1254	0	304	5498
2018-19	2286	1399	2014	698	1021	0	278	5462
2017-18	2235	1392	2000	695	1001	0	262	5314
2016-17	2153	1394	2002	686	981	0	259	5275
2015-16	2056	1395	1971	673	966	0	268	5412
2014-15	2070	1401	1984	684	965	0	256	5612
2013-14	2101	1414	2010	681	958	0	245	6028
2012-13	2129	1399	1942	663	922	0	255	4562
2011-12	2078	1410	1958	657	918	0	278	5298
2010-11	1930	1416	1977	637	889	0	281	5554
2009-10	1798	1392	1892	619	851	0	265	4182
2008-09	1909	1419	1953	636	889	0	249	4386
2007-08	1860	1410	1952	632	881	0	226	4477
2006-07	1756	1398	1924	627	868	0	217	4151
2005-06	1715	1412	1927	608	843	0	203	3977
2004-05	1652	1406	1911	592	811	0	184	4067
2003-04	1727	1407	1897	571	780	0	168	4100
2002-03	1535	1319	1739	539	731	0	161	4830
2001-02	1734	1407	1880	569	784	0	174	4702
2000-01	1626	1413	1853	552	762	0	167	4358
1999-00	1704	1411	1884	575	792	0	181	4620
1998-99	1627	1428	1917	574	787	784	168	4916
1997-98	1552	1420	1900	552	757	760	162	5224
1996-97	1614	1429	1895	551	760	764	143	5611
1995-96	1491	1422	1875	534	714	721	139	6126
1994-95	1546	1430	1881	530	707	709	136	6136
1993-94	1501	1423	1866	513	683	670	124	6365
1992-93	1457	1427	1857	503	668	654	122	7079

1991-92	1382	1416	1822	499	657	647	127	7213
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Source: RBI data on Indian economy.

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Correlations			Collinearity Statistics	
	B	Std. Error				Beta	Zero-order	Partial	Part	Tolerance
β_0	-990.408	1112.225		-.890	.383					
β_1	.510	1.542	.039	.331	.744	.042	.070	.013	.108	9.254
B ₂	-.326	1.134	-.082	-.288	.776	.896	-.061	-.011	.019	53.133
β_3	3.504	1.481	.812	2.366	.027	.972	.450	.092	.013	77.609
β_4	.525	.232	.271	2.258	.034	.944	.434	.088	.105	9.521
β_5	-.077	.073	-.086	-1.063	.299	-.647	-.221	-.041	.232	4.304
β_6	-.243	.737	-.050	-.330	.745	.936	-.070	-.013	.065	15.354
β_7	.040	.018	.120	2.255	.034	-.135	.433	.088	.534	1.872

a. Dependent Variable: Y

Table 2 provides the results of the linear regression and the regression coefficient of Net Irrigated Area in hectare is 3.504 indicating one hectare increase in this input the food grains output increases by 3.5 kg per hectare, while the other regression coefficients give different situation as found from the table.

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-723.706	116.853		-6.193	.000
	Net Irrigated Area	4.195	.191	.972	21.997	.000
2	(Constant)	-487.781	130.784		-3.730	.001
	Net Irrigated Area	2.969	.449	.688	6.617	.000
	Gross Irrigated Area	.593	.201	.307	2.950	.006

a. Dependent Variable: Total Foodgrains

Model		Beta In	t	Sig.	Partial Correlation	Collinearity Statistics
						Tolerance
1	Net area sown	.054 ^b	1.243	.225	.233	1.000
	Gross Sown Area	.128 ^b	1.307	.202	.244	.198
	Gross Irrigated Area	.307 ^b	2.950	.006	.494	.142
	Area under high yield varieties	.043 ^b	.704	.487	.134	.526
	Consumption of Fertilizer (N+P+K) (lakh tonnes)	.064 ^b	.417	.680	.080	.084
	Consumption of Pesticides (Technical Grade Materials) ('000 tonnes)	.103 ^b	2.456	.021	.427	.943
2	Net area sown	.004 ^c	.087	.931	.017	.803
	Gross Sown Area	.023 ^c	.230	.820	.045	.163
	Area under high yield varieties	.012 ^c	.207	.838	.040	.504
	Consumption of Fertilizer (N+P+K) (lakh tonnes)	-.015 ^c	-.108	.915	-.021	.081
	Consumption of Pesticides (Technical Grade Materials) ('000 tonnes)	.080 ^c	2.039	.052	.371	.894

a. Dependent Variable: Total Foodgrains

b. Predictors in the Model: (Constant), Net Irrigated Area

c. Predictors in the Model: (Constant), Net Irrigated Area, Gross Irrigated Area

	Minimum	Maximum	Mean	Std. Deviation	N
Predicted Value	1383.13	2407.34	1831.77	283.024	30
Residual	-132.210	101.902	.000	58.794	30
Std. Predicted Value	-1.585	2.034	.000	1.000	30
Std. Residual	-2.170	1.672	.000	.965	30

a. Dependent Variable: Total Foodgrains

Tables 3 to 5 provides the results of stepwise regression and also endorses the results given in Table 2.

Chart 1: Distribution of total food grains and inputs

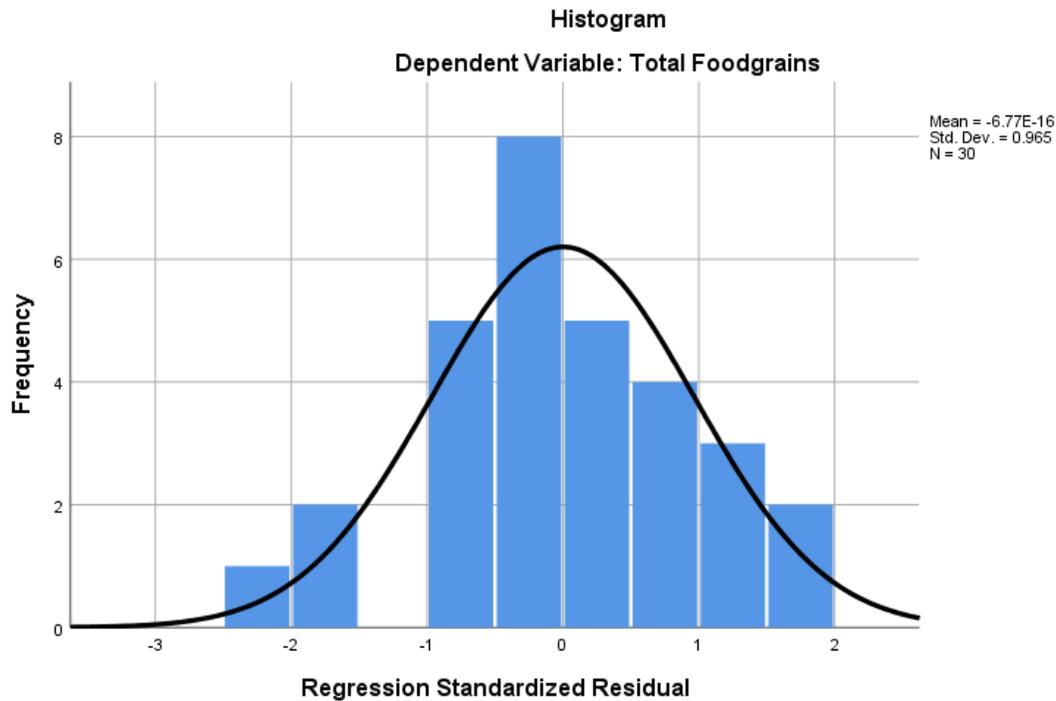


Chart 2: Normal P-P Plot of Regression Standardised Residual

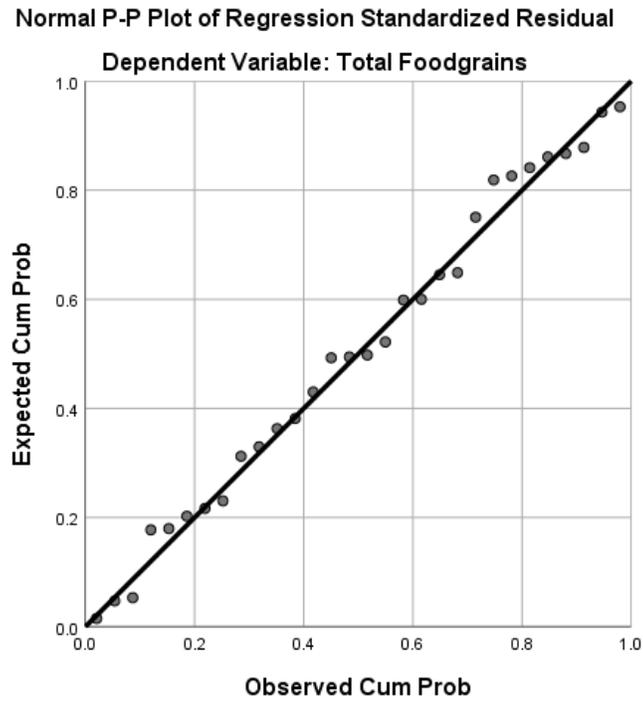


Chart 3: Partial Regression Plot of NIA

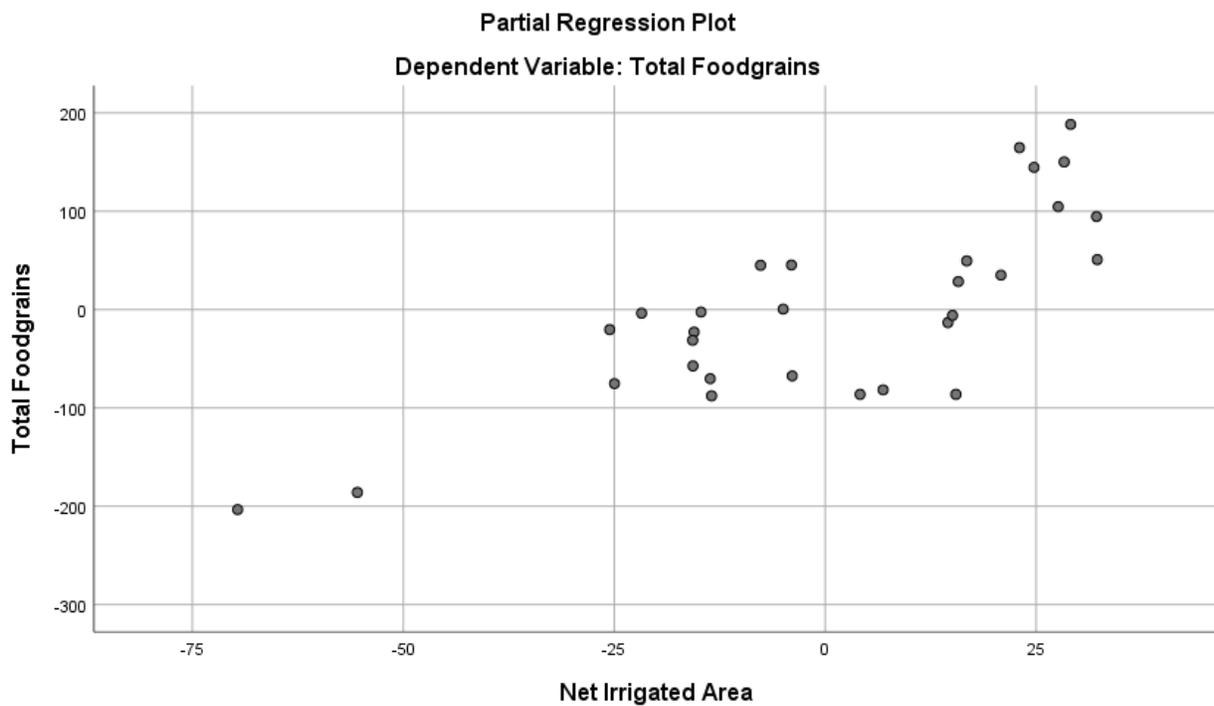
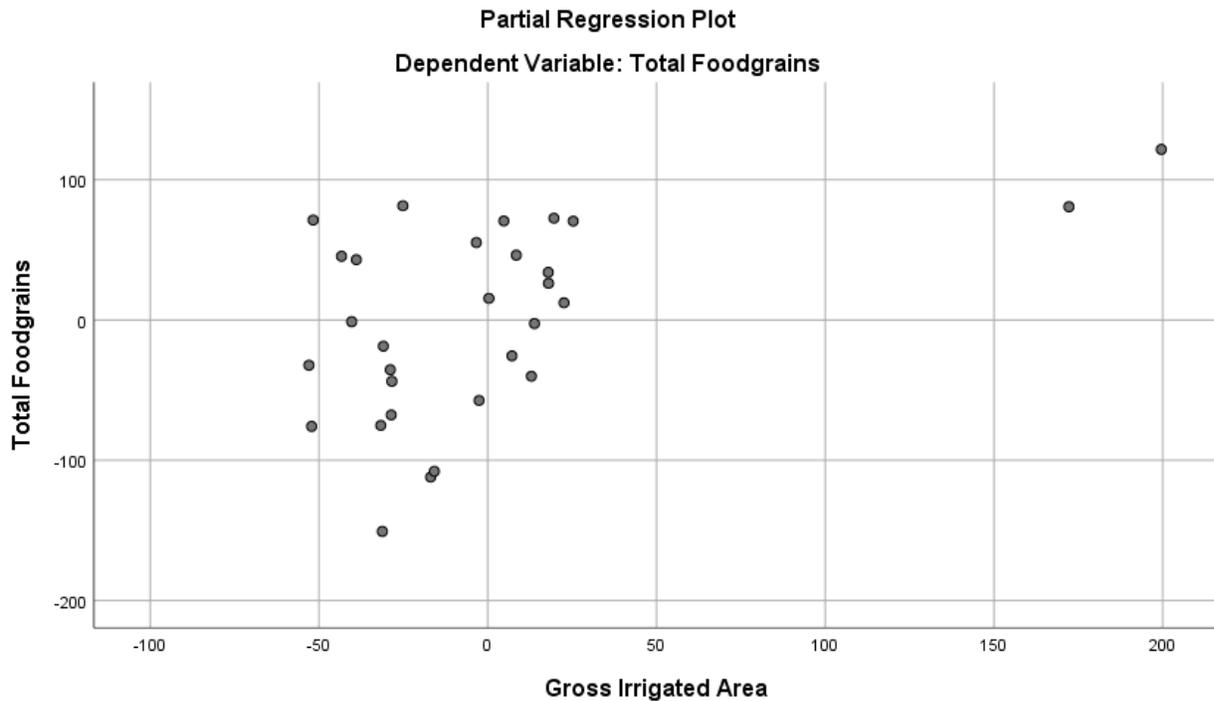


Chart 4: Partial Regression Plot of GIA



Charts 1 to 4 shows the Distribution of total food grains and inputs, Normal P-P Plot of Regression Standardised Residual, Partial Regression Plot of NIA, and Partial Regression Plot of GIA which describes the distribution of data of dependant and independent variables over the periods.

Conclusion

The application of mathematical models in agriculture discussed portrays that the main types of methods of various mathematical means and tools like analytical, simulation and empirical. The mathematical modelling incorporates five stages encompassing formulation, solution, interpretation and validation. Any mathematical model has various components that varies according to its construction and they include variables or decision parameters, constants and calibration parameters, input parameters, data, phase parameters, output parameters, and noise and random parameters. The mathematical model combines equations, charts, formulas etc., and it is formulated on the basis of theory, literature and logical manner. These mathematical models have applications of manifold which provides precision and strategy to provide solution to a problem. Regression models are under mathematical modelling which is widely used in the present-day research.

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