Mathematical Analyses in Genetics and Evolution

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Mathematical Analyses in Genetics and Evolution

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Abstract

Evolution is the phenomenon of the development organisms from more simple earlier forms to more advanced forms in the course of a long period of time. The Genetics refers to inheritance of characters from the parental generation and to the changes in the characters that occur between the offspring of the parents. Mendelian heredity principles forms the foundation over which the modern understanding of the evolutionary process, i.e., "Modern Synthesis" was built. Mathematical derivations, equations and models were used by geneticists and evolutionists as tools to understand the Evolutionary Genetics over the period of time.

Introduction:

Evolution is the change in the characteristics of a species (a group of intra-breeding organisms) over several generations. In other words, it is the history of development (Phylogeny) of organisms from their earlier forms over a period of time. The science of Genetics refers to the heredity and variation. The heredity is the biological process by which the characteristics are transmitted from the parents to the offspring through the agents called Genes. The variation refers the differences that occur in the characteristics among the members of the offspring that are inherited from the same parents. These variations may be outwardly expressed as Phenotypic Variations or inwardly present as Genetic Variations. Therefore, there exists a very close relationship between the studies of Genetics and Evolution. This paved the way to understand the Evolutionary Process through Genetics and in particular the Modern Genetics. In the present paper the mathematical applications to understand the principles of Genetics and Evolutions particularly with reference to Mendelian inheritance, Hardy-Weinberg Law and Genetic Drift are discussed.

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Mathematics of Mendelian Inheritance:

Gregor Johann Mendel, an Austrian Monk, could prove for the first time the principles of Genetics through his famous studies on the growth pattern of the Pea plants with simple mathematical derivations. The four Mendel's Laws of Unit Characters, Dominance, Segregation, and Independent Assortment are explained with simple phenotypic and genotypic arithmetic ratios. The respective Mendelian ratios for the outward expression of characters (Phenotypes) of Monohybrid and Dihybrid crosses are 1:3 and 9:3:3:1 while the inner genetic compositions of these crosses are 1:2:1 and 1:2:1:2:4:2:1:2:1 respectively. Test Cross is a genetic technique used to find out the real inner genetic composition of outward expression of characters. The simple mathematical ratios for Monohybrid and Dihybrid test crosses are 1:1 and 1:1:1:1 respectively.

The symbolism used in Mendelian equations to explain the hybrid crosses are of the nature of Algebraic structures. Seventy-four years after the Mendel's work the application of formal language of abstract algebra to study the genetics was brought to light by Etherington.

Hardy-Weinberg Equilibrium Equation:

The Genes for the manifested characters are represented by genetic units called alleles which are found in the chromosomes. These alleles are generally found pairs, one member inherited from the male parent and the other from the female parent. Among these alleles one may be dominant in showing outward expression under either homozygous (AA) or heterozygous (Aa) conditions The other allele is recessive that could express only under recessive condition (aa). Hardy-Weinberg Equilibrium Equation was formulated by Hardy and Weinberg in 1908 to know the frequencies of the alleles in a population. The mathematical derivation of a monohybrid (with only single set of dominant and recessive alleles) the Hardy-Weinberg Equilibrium is as follows:

'p' = all the alleles of the dominant gene found in homozygous condition **(AA)** and half of the number of alleles found in heterozygous condition **(Aa)**

$$\mathbf{p = AA + 1/2Aa}$$

and

'q' = all the alleles of the recessive gene found in homozygous condition **(aa)** and half of the number of alleles found in heterozygous condition **(Aa)**

$$\mathbf{q = aa + 1/2Aa}$$

As the frequency of one plus the frequency of the other equals 100% of the alleles

$$\mathbf{p + q = 1}$$

Then the logic derivation is:

$$\mathbf{p = 1 - q \text{ and } q = 1 - p}$$

$$\mathbf{(p + q)^2 = 1}$$

Then the binomial expansion formula is: $\mathbf{P^2 + 2pq + q^2 = 1}$

This Equilibrium formula facilitates us to predict the frequency of the alleles in a population whether they are outwardly expressed or not.

Hardy-Weinberg Equilibrium Equation was formulated based on the assumptions like the population in question is very large, mating between individuals are at random and not through selection, no immigration or emigration of individuals with the addition or loss of alleles, no mutation of the genes. This Equilibrium concept explains the expected frequency of the alleles of such populations that are not evolving due to the absence of selection pressure from the Nature. But in reality, deviations occur from this Equilibrium due to the changes in the above assumptions which cause Evolutionary changes in populations. Hence, the application of this Equilibrium is more of the nature of Null Hypothesis.

To generalize the Hardy-Weinberg Equilibrium, Bernstein introduced Bernstein algebras as a measure to easily understand the principle of Hardy–Weinberg law, which states that a randomly mating population is genetically in equilibrium state.

<u>Genetic Drift or Sewell Wright Effect:</u>

The deviations in the Hardy-Weinberg Equilibrium of a population will cause changes in evolutionary process and in frequencies of genes. Genetic Drift or Sewell Wright Effect or Founder Effect is one such known deviation that occurs in smaller populations. Random and selective mating pressures among the individuals are the deciding factors in the fate of allelic pairs (genes) in a population. In Genetic Drift there may be selective mating pressures between individuals. This results in either elimination of some of the genetic traits or fixation of some other genetic traits in due course of the evolution of the population.

Different mathematical models with algebraic equations were designated to understand the process of Genetic Drift and evolutionary changes in the gene frequencies. Equation suggested by Barton and his associates is as follows.

$$V_t \approx pq(1 - \exp(-t / 2N_e))$$

Assuming genetic drift is the only evolutionary force acting on an allele, after t generations in many replicated populations, starting with allele frequencies of p and q, the variance in allele frequency across those populations will be as per the above equation.

Wright-Fisher model is another equation formulated based on the assumptions that the allelic pairs of a gene, **'p'** and **'q'** found in '***N'*** number of individuals of the population either as homozygous or heterozygous conditions and the generations are not overlapping. Under such condition each copy of the gene found in the new generation is drawn independently at random from all copies of the gene in the old generation. The equation of this model to calculate the probability of obtaining ***'k'*** copies of an allele that had frequency ***'p'*** in the last generation is as follows:

$$\frac{(2N)!}{k!(2N-k)!} p^k q^{2N-k}$$

Where the symbol "**!**" signifies the factorial function.

A still other model, **Moran model** is based on the overlapping generations. At each generation, one individual is chosen to reproduce and one another individual is chosen to die. So in each generation, the number of copies of a given allele can go up by one, go down by one, or can stay the same. The linear algebraic matrix for this model is tri-diagonal which is easier for this model.

Conclusion:

Carl Friedrich Gauss considered mathematics as the "Queen of Sciences". The symbols, algebraic derivations and the practically and theoretically oriented Number Theory of mathematics uncover the nature of physical and biological realities. Charles Darwin believed that the Evolution occurs because of the Natural Selection of the organisms with favoured variations. His theory of evolution with Mendelian hereditary principles in a joint mathematical framework forms the basis of 20th Century understanding of the evolutionary process. This modern idea of evolution is called "Modern Synthesis" by evolutionary biologist Julian Huxley in1942.

As an important fundamental principle of population genetics, the Hardy-Weinberg Law explains that the frequencies of the genotypes are constant between generations in a population in the absence of interferences by outside factors. This Equilibrium phenomenon can be used as a measure of Null Hypothesis to prove whether Evolution is taking place in a population.

Genetic drift is the deviating principle from Hardy-Weinberg Law which occurs in a population of smaller size. In this drift the selective matting pressure causes in a population the elimination of some genetic traits or stabilization of some other genetic traits. The algebraic equations of Barton, Wright-Fisher and Moran models are used as tools to explain the phenomenon of Genetic Drift in a clearer and easier way.

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