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Differential Calculus and its Application in Economics- A Study with Reference to Consumer Demand Theory

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Abstract

Calculus is one of the important components of mathematical tools used in economics. This enables understanding, improving and problem-solving tools for economic variables. The mathematical analysis contains differential calculus and integral calculus. Calculus is mostly expressed in functions and derivatives. The two-consumer demand theories are cardinal and ordinal. The former is the marginal utility approach and the latter is indifference curve analysis popularised by authors like Gossen (1854), William Jevons (1871), Leon Walras (1874), Carl Marshall (1890), Menger (1950), Hicks (1956), Pareto (1909), P.A. Samuelson (1949), and Robbins (1984) etc. These models analysed the relationship between the price of a commodity and the quantity demanded of the same commodity for deriving individual and market demand curves. The coefficient of price, income and cross elasticities and price, income and substitution effects are also part of these theories.

Keywords: Calculus, mathematical economics, reverse progression, consumer demand theory, elasticity of demand.

Introduction

Differential calculus is widely used in economics. In mathematical economics, calculus deals with finding the different properties of integrals as well as derivatives of functions from mathematics. It is based on the summation of the infinitesimal differences. Calculus is the study of the continuous or rate of change of a function. This is helpful for easily understanding the concepts of a subject and also for improving problem-solving skills. Function, limit, continuity,

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derivatives, and integration have been commonly studied in calculus. It has two major branches and those two fields are related to each other by the fundamental theorem of calculus viz., (1) Differential calculus and (2) Integral Calculus. These two branches of calculus lay the foundation for a higher level of mathematical analysis.

The differential is a calculus material that studies a derivative function. The derivative function should be differentiable and differentiation is the derivative process of finding the derivative of a function. It deals with changes in function variables to solve problems related to change in the form of independent variables and dependent variables. That is the behaviour and rate of change are studied in differential calculus. The differential calculus is applied in economics to demand and supply models, compute marginal cost, marginal revenue, maxima and minima, elasticities, partial elasticities, profit maximization, loss minimization, etc.

Integral calculus is a reverse progression of differential and also it is called derivatives. Differentiation is a reversal of integration. As a result, integration is an inverse process of differentiation. Integration is the process to find the antiderivative of a function and hence integration may be an inverse process of differentiation. In the formulation of subproblems, the application of derivatives is helpful to define the response of a problem for a solution.

Calculus is mostly expressed in functions and derivatives. Derivatives are imperative in finding the solution to a problem, whereas functions deal with two or more variables which are normally symbolized by using X and Y. The values of Y and X emphasize the functional relationship. The derivative deals with the rate of change of a function relating to its value.

In an equation with an unknown function say, $y=f(x)$, where its derivative is a differential equation, and by solving this type of equation the user gets evidence about how the dependent variable (Y) and independent variable (X) provide insight into the situation for which it is formed. The unknown function may be expanded with many dependent and independent variables. Economic theories widely use it for better and easy explanation and also to quantify the relations as attempted in this study to analyse nuances of 'differential calculus and its application in Economics concerning consumer demand theory.

Review of Literature

Sanjay Tripathi (2020) concluded that integral calculus helps to explain supply and demand theory easily. The changes in the price determine the quantity of supply and demand of a commodity and also establishes its inverse or positive relation among these variables. He found 'first and second derivatives are useful in optimization problems which determine the maximum profits. In short, we say that the application of calculus to business and economics, such as maximum and minimum problems, marginal analysis, and cost analysis can be computed more easily.'

Olshavsky and Granbois (1979) have discussed that the major proportion of our purchases involve no decisions at all. Anton P. Barten and Volker Böhm, (1982) have discussed that consumer theory is about the demand for commodities that has alternative assumptions on the behavioural rules of the consumer and the constraints in decision making. The traditional model deals with the preference of a consumer over alternative bundles. Commodities consist of goods and services. The commodity has physical characteristics, location, and time of its availability. They have concluded that 'In studies of behaviour under uncertainty, an additional specification of the characteristics of a commodity relating to the state of nature occurring is added, which leads to the description of a contingent commodity. The behavioural rule consists of the maximization of these preferences under a budget restriction, which determines the trading possibilities. The principal results of the theory consist of the qualitative implications on observed demand of changes in the parameters, which determine the decision of the consumer. The historical development of consumer theory indicates a long tradition of interest of economists in the subject, which has undergone substantial conceptual changes over time to reach its present form.'

Clements, Selvanathan and Selvanathan (1995) have discussed the demand function of a commodity and how changes in price and income, derivation of testable hypotheses with theoretical restriction, Slutsky symmetry, utility function, utility maximization, indirect utility function, cost function and application of differential approach. Ennio Bilancini and Leonardo Boncinelli's (2010) study found that 'if preferences are rational, strictly monotone and continuous, then non-strict convexity implies the existence of an upper contour set that is not strictly convex and that satisfies the assumptions of Lemma 2, Lemma 3. At this point, we apply Lemma 1 and we obtain a hyperplane supporting the upper contour set at multiple points.

Jonathan Levin and Paul Milgrom (2004) have explained that "consumer theory is concerned with how a rational consumer would make consumption decisions. What makes this problem worthy of a separate study, apart from the general problem of choice theory, is its particular structure that allows us to derive economically meaningful results. The structure arises because the consumer's choice sets are assumed to be defined by certain prices and the consumer's income or wealth."

Grewal et al. (2013) have analysed the impact of social, mobile and in-store shopper marketing practices on pre-purchase, purchase and post-purchase practices. Wolny and Charoensuksai (2014) have elaborated the decision-making model with a consumer-centric diary-based methodology inductively concerning multi-channel shopping. Rachel Ashman, Michael R. Solomon, and Julia Wolny (2015) have analysed Engel, Kollatt, and Blackwell's (EKB) decision-making model and its impact on individual decision-making. They discussed the impact of the rise of participatory culture on individual decision-making. It also explored the relevance of current participatory online culture as it is highly influenced by the decision-making process coupled with social collective variables by including the 'old EKB model, with some minor tweaks, which still provides valuable insights into and explanations of consumer decision-making. A conceptual analysis of the decision-making stages has specific ramifications for extant theory and in particular the EKB model.'

Svetlana Sazanova (2020) studied the theory of consumer behaviour in economics concerning the ratio of rational and irrational motives of behaviour, economic communications, rational reconstruction of scientific knowledge, comparative analysis, scientific abstraction etc. This study also covered the areas of economic thought in a historical context, with ancient philosophers, scholastics, mercantilists, classical political economy, neoclassical economic theory, behavioural economics, institutional economics, and systemic economic theory. The author concluded that the theory of consumer behaviour is a synthesis of the theory of productive consumption and the theory of economic communications.

Vasily E. Tarasov (2020) has explained the Slutsky equation, “let us assume that there are only two goods (x and y). In microeconomics, two types of demand functions are used: the compensated demand function, xc (px, py, U), and the ordinary (uncompensated) demand function, x (px, py, I). The compensated (Hicksian) demand function describes the demand of a consumer over a bundle of goods (x and y) that minimizes their expenditure while delivering a fixed level of utility. The compensated demand functions are convenient from a mathematical point of view since these functions do not require income or wealth to be represented. In addition, the function xc (px, py, U) is linear in (x, y), which gives a simpler optimization problem. Unfortunately, these functions are not directly observable. The uncompensated (Marshallian) demand functions x (px, py, I) are convenient from an economic point of view. However, this convenience is due to the fact that the uncompensated demand function x (px, py, I) describes demand given prices px, py and income I that are easier to observe directly in economics. The compensated (Hicksian) demand function is defined by the equation xc (px, py, U) = x (px, py, E, px, py, U).”

Differential Calculus and Consumer Demand Theory

The consumer demand theory having rationality as the common assumption has two basic approaches viz., cardinal (classical theory) and ordinal (indifference curve approach). The former is the marginal utility approach and the latter is indifference curve analysis. These approaches have been popularised by authors like Gossen (1854), William Jevons (1871), Leon Walras (1874), Carl Marshall (1890), Menger (1950), Hicks (1956), Pareto (1909), P.A. Samuelson (1949), and Robbins (1984) etc. The consumer purchases a good as he gets satisfaction while possessing the good. The consumer also derives utility from the possession of the good. The utility approach has multivarious forms i.e., independent, interdependent and additive utilities. These economists have tried to measure the utility or satisfaction derived by the consumer. Given the basic understanding of the subject, it is attempted to analyse Marginal utility (MU) which sets the beginning of consumer demand theory.

Marginal utility (MU) is addition to total utility for a unit increase in the consumption of the good or increment in the total utility (TU) is MU. Calculus helps to calculate marginal utility easily as given by leading books in microeconomics (Richard A. Bilas, 1972 and K.C. Roychowdhury, 1991) as derived below:

Utility function (U)

$U=F(X,Y)$

Where X and Y are two goods, and it is constrained by

$XP_X + YP_Y = I$

Where X and Y are the numbers of units of commodities X and Y purchased for the respective prices $P_X + P_Y$ for the given income I

$F(X, Y, \lambda) = f(X.Y) + \lambda (I - XP_X - YP_Y)$

Where λ is the Lagrange Multiplier. The first-order conditions hold good for maximization,

$\partial F / \partial X = f_X - \lambda P_X = 0$
$\partial F / \partial Y = f_Y - \lambda P_Y = 0$
$\partial F / \partial \lambda = I - XP_X - YP_Y = 0$

And $\lambda = f_X / P_X$, $\lambda = f_Y / P_Y$

Hence, $f_X / P_X = f_Y / P_Y$

Where $f_X = MU_X$ and $f_Y = MU_Y$

Here λ is the marginal utility of money (MUm) and is found by $\partial F / \partial I = \lambda$. Using this method, it is evident that it is not necessary to have diminishing marginal utility to derive a demand curve which will be negatively sloped provided X and Y are interdependent. This forms the basis for deriving individual demand curve which facilitates the market demand curve.

Let us consider, $U = X^3 Y^3$

then

$\partial U / \partial X = 3 X^2 Y^3 = MU_X$ and $\partial U / \partial Y = 3 Y^2 X^3 = MU_Y$

X increases so does MU_X and similarly, as Y increases so does MU_Y (Note that MU_X depends on Y quantity and MU_Y depends on X quantity)

$MU_X / P_X = MU_Y / P_Y$

This easily explains the consumer equilibrium.

Hence,

$3 X^2 Y^3 / P_X = 3 Y^2 X^3 / P_Y$ and $YP_Y = XP_X$

Therefore, $XP_X + YP_Y = I$ and $2\ XP_X = I$ or $X = I / 2P_X$

Showing a negatively sloped demand curve with ep=1.

Where ep is the price elasticity of quantity demanded.

Conclusion

The consumer demand theories as propounded by cardinalist and ordinalist schools in different periods have been explained with a simple mathematical model. They preferred the partial equilibrium approach of Marshall or the general equilibrium approach of Walras. The crux of these models is to analyse the relationship between the price of a commodity (Px) and the quantity demanded of the same commodity (Qdx) with the respective assumptions. It is used for deriving individual as well as market demand curves. The coefficient of price, income and cross elasticities of demand determines the commodity as a substitute or complementary. The price, income and substitution effects are also part of these theories. Altogether, calculus helps to explain the consumer demand theory apart from diagrammatical and numerical methods to enhance its scientific validity.

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